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Quantum fluctuations in low-dimensional easy-plane spin models

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Abstract. A self-consistent harmonic approximation is used to treat the low temperature limit of the one and two dimensional $S = 1$ easy plane magnets. For the one dimensional (1D) model, we calculate the gap caused by the presence of an external magnetic field applied in the easy-plane. The quantum phase transition of the one-dimensional model at $T = 0$ is also studied. For the two-dimensional case, the Kosterlitz-Thouless (KT) transition temperature as a function of a single-ion anisotropy term is calculated. The line ends at a quantum critical point, where the KT temperature goes to zero.

PACS. 75.10.Jm Quantized spin models – 75.40.-s Critical-point effects, specific heats, short-range order

1 Introduction

Magnetic systems with reduced dimensionality have attracted much attention in condensed matter physics and provided a basis for numerous insights into the varying roles that quantum and thermal fluctuations play in driving phase transitions. These low-dimensional magnets, because of the enhanced quantum fluctuations, reveal far richer physical behavior than their more conventional three dimensional (3D) counterparts [1]. In fact, the quantum nature and behavior of magnetic systems have, occasionally, surprised the scientific community with features that could never be imagined by using analogies to the corresponding classical systems. Among these interesting features is the behavior of the one dimensional (1D) Heisenberg antiferromagnet which has a qualitative different behavior depending on whether the spin is integral or half integral [2]. For half spins, the 1D Heisenberg antiferromagnet is known to have a gapless ground state while, for integer spin, there is a gap between the first excited state and the ground state. This state can be destroyed by terms modifying the symmetry of the Heisenberg model as, for example, exchange anisotropy and single-site anisotropy.

The spin half 1D antiferromagnet has also a peculiar behavior when subjected to an external magnetic field. The effect of a magnetic field applied to classical 1D antiferromagnets is a magnetization developed by a homogeneous canting of the spins in the direction of the field; the so called spin-flop phase. However, for $S = 1/2$ 1D antiferromagnets in an external magnetic field, the spin chain develops soft modes [3] at the incommensurate wave vectors which connect the field-dependent Fermi points. The behavior of this quantum 1D system can be understood by mapping the spin chain to a 1D system of interacting fermions known as spinons [4]. Classical 1D spin systems do not show incommensurate spin correlations, but they may be a general feature of 1D quantum spin chains [5]. The first experimental evidence for incommensurate spin fluctuations in a $S = 1/2$ antiferromagnetic chain was found by Dender et al. [6] by using the neutron scattering technique on copper benzoate ($Cu(C_6D_5COO)_2 \cdot 3D_2O$). Those authors were able to show that, as predicted by theory, soft modes occur for wave vectors $\tilde{q} = \pi \pm \delta\tilde{q}$ where $\delta\tilde{q} \approx 2\pi M(H)/g\mu_B$, with $M(H)$ being the magnetization per spin. However, the modes are not completely soft because the field also induces an unexpected excitation gap $\Delta(H)$ which depends on the applied field as $\Delta(H) \propto H^{0.65}$. The mechanism of the field induced gap was discussed by Oshihawa and Affleck [7]. They argued that, due to an alternating g-tensor existing in the copper compound, a uniform applied field produces an effective staggered field on the spin chain. Using standard scaling arguments and numerical calculation for the model, Oshihawa and Affleck obtained $\Delta \propto H^{0.67}$, which is in excellent agreement with the experimental result.

The XY model also represents a particularly important example of magnetic systems, impacting problems in several subjects in condensed matter. In two dimensions, this model shows an unusual phase transition, the so-called Kosterlitz-Thouless (KT) transition [8], which has a high temperature massive phase with exponential decay of correlations, and a low temperature massless phase where the correlations decay like a power of the distance [9]. Despite the simplicity of the arguments suggesting a KT transition in a variety of systems, a rigorous proof of the KT nature of the phase transition in many physically

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interesting systems is still lacking. There are also very interesting aspects to be investigated in the 1D XY model. In contrast to the Heisenberg ferromagnet, the ground state of the 1D XY model is no longer trivial [10]. In fact, it shares many properties with the antiferromagnetic 1D Heisenberg model [11]. The low-lying excitation of the 1D XY spin-1/2 model, just as the Heisenberg 1D antiferromagnet, are also the spin 1/2 objects called spinons, quite different from standard spin waves [12]. However, for $S > 1/2$, we can use spin-waves techniques to treat the model. For the XY model with $S > 1/2$, a convenient representation for the spin operators that exploits the symmetry of the model is the one proposed by Villain [13].

In this paper, we start by considering a general Hamiltonian that, for a proper choice of parameters, can describe several models of interest, ranging from the symmetric Heisenberg to the pure XY model. This Hamiltonian is defined as

$$\mathcal{H} = -J \sum_{\mathbf{r},\mathbf{a}} \left[S^x_{\mathbf{r}} S^x_{\mathbf{r+a}} + S^y_{\mathbf{r}} S^y_{\mathbf{r+a}} + \lambda S^z_{\mathbf{r}} S^z_{\mathbf{r+a}} \right] + D \sum_{\mathbf{r}} (S^z_{\mathbf{r}})^2 - g\mu_B H \sum_{\mathbf{r}} S^x_{\mathbf{r}}, \quad (1)$$

where the summations run over all spin sites $\mathbf{r}$ and $\mathbf{a}$ denotes the nearest neighbors to each spin site. We will be interested in 1D and 2D systems with spin $S = 1$. Our discussions will consider D positive ($0 \leq D < \infty$), but the exchange planar anisotropy will vary only in the $0 \leq \lambda < 1$ range. The effect of a magnetic field H applied along the x-direction will be considered only for one-dimensional systems.

As it is well known [14], Hamiltonian (1) can be mapped onto the following Hamiltonian

$$\mathcal{H}_1 = J \sum_{\mathbf{r},\mathbf{a}} \left[S^x_{\mathbf{r}} S^x_{\mathbf{r+a}} + S^y_{\mathbf{r}} S^y_{\mathbf{r+a}} - \lambda S^z_{\mathbf{r}} S^z_{\mathbf{r+a}} \right] + D \sum_{\mathbf{r}} (S^z_{\mathbf{r}})^2 - g\mu_B H \sum_{\mathbf{r}} (-1)^r S^x_{\mathbf{r}}, \quad (2)$$

indicating that the spectrum of the original Hamiltonian is invariant under the transformation $(J, \lambda) \rightarrow (-J, -\lambda)$, where, now, λ varies in the $-1 < \lambda < 0$ range, and provided that the magnetic field H is transformed into a staggered field. In order to apply the invariance of the spectrum into a corresponding statement for the transverse correlation function, it is necessary to shift the Brillouin zone by $k \rightarrow k + \pi$. With this in mind, we will assume J positive in (1) but, when convenient, will make comparisons to the antiferromagnetic model.

In Sections 2 and 3 of this work, we discuss the one-dimensional model. The gap induced in the 1D antiferromagnet by the staggered magnetic field is analysed in Section 2 by using the self-consistent harmonic approximation (SCHA). The quantum phase transition of the 1D XY ferromagnet with single- site anisotropy ($\lambda = H = 0$) is the subject of Section 3 where we evaluate the critical value for the anisotropy parameter, D_c. The SCHA is also used to study the two-dimensional (2D) XY ferromagnet. In Section 4, we obtain the Kosterlitz-Thouless critical temperature, T_{KT}, as a function of the single-site anisotropy D. The phase transition terminates at a quantum critical point given by D_c. This critical anisotropy depends on the spin S value, and this dependence is shown in Section 4. We also compare the behavior of T_{KT} as a function of the anisotropy for the classical XY and planar rotor models showing that as $D \rightarrow \infty$ the two models coincide. Our conclusions are presented in Section 5.

2 One dimensional case

No exact solution of the Hamiltonians given by equations (1) or (2) is available for $S > 1/2$. An analytical study of the model described by (2) with $H = 0$ has been presented recently by Kennedy and Tasaki [15]. Those authors studied the phase diagram of $S = 1$ antiferromagnetic chains with particular emphasis on the physical properties of the massive phases. They proved the existence of the Haldane phase for a particular class of Hamiltonians that, unfortunately, does not include the usual Heisenberg Hamiltonian. Papanicolaou and Spathis [14] studied the 1D ferromagnet described by (1), also in the absence of magnetic field, by using semiclassical and strong-coupling methods. They obtained that the model's ground state is XY-like for small values of D ($0 \leq D < J$), but, for large D values ($D \gg J$), we can expect a remarkably different ground state. In fact, they showed that the energy-momentum dispersion develops a mass gap for $D > J$; there is a gapless-gapped Kosterlitz-Thouless transition between the XY phase and the large-D phase. This behavior was also observed by Chen et al. [16] in their study of the 1D antiferromagnet with $H = 0$. Using numerical exact diagonalization of finite size systems, Chen et al. [16] were able to obtain the phase diagram of the model with a rich variety of phases labelled as Haldane, large D, XY and ferromagnetic Ising phases. In particular, they showed that for the antiferromagnet with $-1 < \lambda \leq 0$ and $\delta = D/J < 0.44$ the ground state has gapless phase, in accordance with the result found by Papanicolaou and Spathis.

In this work, we will study Hamiltonian (1), considering $H \neq 0$, by using the self-consistent harmonic approximation. The SCHA was originally proposed for the 2D classical planar rotor [17], extended later by Menezes et al. [18] to the classical XY model and by Pires [19] to the quantum XY model. The approximation consists in replacing the Hamiltonian of a system by an effective harmonic Hamiltonian with temperature dependent renormalized parameters. In order to apply the SCHA to Hamiltonian (1), we start by writing the spin components in terms of the Villain representation [13],

$$S^+_n = e^{i\phi_n} \sqrt{(S + 1/2)^2 - (S^z_n + 1/2)^2},$$
$$S^-_n = \sqrt{(S + 1/2)^2 - (S^z_n + 1/2)^2} e^{-i\phi_n}, \quad (3)$$

where n denotes the spin sites of the magnetic chain. There is an extensive literature describing the SCHA [17–20], and, for this reason, we will only sketch the main steps leading to the expressions for the temperature dependent parameters. Inserting (3) in (1), and following the procedure described, for example, in reference [20], we arrive at the following effective Hamiltonian

$$\mathcal{H}_o = J\sum_n \left[\frac{\tilde{S}^2}{2}\rho\left(\phi_{n+1}-\phi_n\right)^2 + (S_n^z)^2 - \lambda S_n^z S_{n+1}^z\right] + D\sum_n (S_n^z)^2 + \frac{g\mu_B HS}{2S^2}\sum_n (S_n^z)^2 + \frac{g\mu_B HS}{2}\gamma\sum_n \phi_n^2, \tag{4}$$

where we have used $\tilde{S}^2 = S(S+1)$. The parameters ρ, known as stiffness, and γ are the SCHA temperature dependent parameters defined as

$$\rho = \left\langle \left[1-\left(\frac{S_n^z}{\tilde{S}}\right)^2\right]\right\rangle e^{-\frac{1}{2}\langle(\phi_{n+1}-\phi_n)^2\rangle} \tag{5}$$

$$\gamma = e^{-\frac{1}{2}\langle\phi_n^2\rangle}\left\langle\left[1-\frac{1}{2}\left(\frac{S_n^z}{\tilde{S}}\right)^2\right]\right\rangle. \tag{6}$$

Here, $\langle\cdots\rangle$ means a thermal average that must be taken using the quadratic Hamiltonian $\mathcal{H}_o$ given by (4). Equations (5) and (6) are self-consistent equations that must be solved iteratively to give the stiffness ρ, and γ for each temperature.

Taking the Fourier transform of (4), defining $\delta = D/J$, $h = g\mu_B H/(2J\tilde{S})$, and setting $J = 1$ to fix the energy scale, we obtain

$$\mathcal{H}_o = \sum_q \Big\{\tilde{S}^2\left[\rho(1-\cos q)+\gamma h\right]\phi_q\phi_{-q} + \left[1+\delta-\lambda\cos q + h\right]S_q^z S_{-q}^z\Big\}. \tag{7}$$

Hamiltonian (7) can be diagonalized by a Bogoliubov transformation and we obtain the dispersion relation as given by

$$\omega_q = 2J\tilde{S}\left\{\left[\rho(1-\cos q)+\gamma h\right]\left[1+\delta-\lambda\cos q+h\right]\right\}^{1/2}. \tag{8}$$

The correlation functions $\langle\phi_q\phi_{-q}\rangle$ and $\langle S_q^z S_{-q}^z\rangle$, needed in the calculation of equations (5) and (6), can also be obtained, giving

$$\langle\phi_q\phi_{-q}\rangle = \frac{1}{2\tilde{S}}\sqrt{\frac{1+\delta-\lambda\cos q+h}{\rho(1-\cos q)+\gamma h}}\ \coth\left(\frac{\beta\omega_q}{2}\right), \tag{9}$$

$$\langle (S_q^z)^2\rangle = \frac{\tilde{S}}{2}\sqrt{\frac{\rho(1-\cos q)+\gamma h}{1+\delta-\lambda\cos q+h}}\ \coth\left(\frac{\beta\omega_q}{2}\right). \tag{10}$$

Inserting equations (8–10) into equations (5) and (6), we can obtain several quantities of interest as, for example, the gap $\Delta = \omega(q=0)$ as a function of the applied field. In Figure 1, we show the result obtained for the gap Δ as a function of h for $\lambda = \delta = 0$ and, in Figure 2, the gap as a function of the temperature for $h = 0.10$ and $\lambda = \delta = 0$. A fit to the curve shown in Figure 1, gives $\Delta = 1.63h^{\alpha}$ with $\alpha = 0.67$, which is approximately 2/3. It must be noticed that the standard spin-wave approximation, i.e., equation (8) with $\rho = \gamma = 0$ predicts a gap proportional to $h^{1/2}$. We thus conclude that the 1D quantum fluctuations are responsible for the change of behavior from $h^{1/2}$ to $h^{0.67}$. At the present time, we are not aware of any theoretical calculation predicting the behavior of the gap as a function of the applied magnetic field for the quantum XY model.

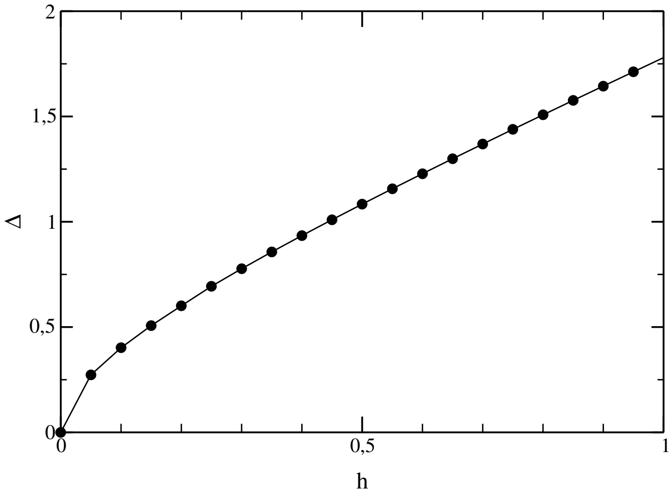

Fig. 1. Gap Δ as a function of the magnetic field h, according to Hamiltonian (2). The calculation was performed inserting equations (9–11) into equations (5) and (6) for zero temperature. We have used $J = S = 1$ and $\delta = 0$.

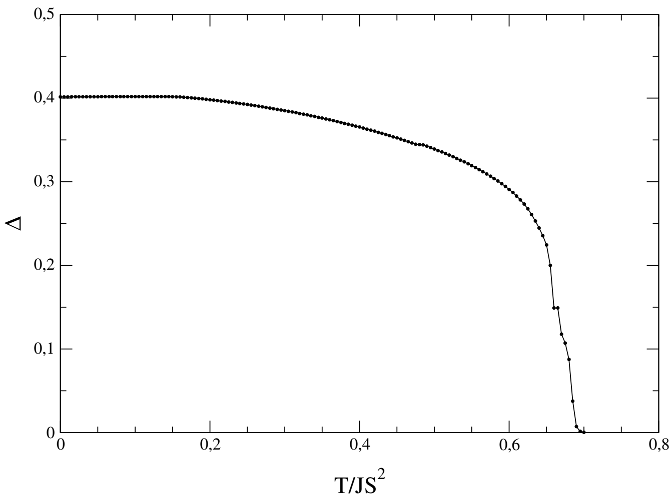

Fig. 2. The gap induced by the staggered field in (2) as a function of the temperature. As in Figure 1, we used $J = S = 1$ and $\delta = 0$. The result displayed was obtained for $h = 0.10$.

As we can see in Figure 2, the gap decreases monotonically with increasing temperature, but around $T_2/J \approx 0.65$ it drops discontinuously to zero. This discontinuous change is an artifact of the SCHA, but for $T < T_2$ the SCHA is believed to describe the system correctly.

3 Quantum-phase transition in the 1D model

Quantum phase transitions (QPT) have attracted much interest in recent years [21,22]. These transitions take place at the absolute zero of temperature, where crossing the phase boundary means that the quantum ground state of the system changes in some fundamental way. This is accomplished by changing not the temperature, but some parameter in the Hamiltonian. One way to study QPT is by means of the path-integral formalism where a d-dimensional quantum system at $T = 0$ is transformed into a classical $d + 1$ dimensional model. We will use this technique here in order to study the QPT of a 1D system described by the Hamiltonian

$$\mathcal{H}_2 = -J \sum_n \left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) + D \sum_n (S_n^z)^2. \quad (11)$$

which corresponds to the Hamiltonian given by (1) for zero magnetic field and $\lambda = 0$. The ground state of this Hamiltonian for $D \to \infty$ corresponds to the large-D phase, where each quantum spin is restricted to the state $S^z = 0$. For $D = 0$, we must have an $XY-$like ground state: thus, there exists a critical value for this anisotropy, D_c, defining the QPT.

The continuum limit of Hamiltonian (11) can be taken, precisely, by using the Villain's transformation defined in (3) and using the relation [23]

$$\cos\phi = e^{\langle \phi^2 \rangle/2} \left[1 - \frac{\phi^2}{2} + \cdots \right]. \quad (12)$$

This procedure allows us to write Hamiltonian (11) as

$$\mathcal{H}_2 = J \sum_n \left[\tilde{\delta}(S_n^z)^2 - \tilde{S}^2 \cos(\phi_n - \phi_{n+1}) \right], \quad (13)$$

where we have defined $\tilde{\delta} = \rho + D$, and ρ was defined in (5). Now, using the equation of motion for ϕ,

$$\dot{\phi}_n = -i[\phi_n, H] = -2i\tilde{\delta} S_n^z, \quad (14)$$

we obtain the continuum limit of (11) which is given by,

$$\mathcal{H}_2 = \frac{J}{2} \int dz \left[\frac{1}{2\tilde{\delta}} \left(\frac{\partial \phi}{\partial t} \right)^2 + \rho \tilde{S}^2 \left(\frac{\partial \phi}{\partial z} \right)^2 \right], \quad (15)$$

which is the $\mathcal{O}(2)$ non-linear sigma model.

Boschi et al. [24] using a semiclassical approximation, and making use of spin coherent states, obtained the same Hamiltonian given by equation (15) with $\rho = 1$. We remark that, for integer spin S, the Berry phases can be ignored.

In the path integral approach, we re-scale the temporal and spatial coordinates according to

$$x = -i\sqrt{2\tilde{\delta}}t, \qquad y = z/(\tilde{S}\sqrt{\rho}) \quad (16)$$

and, taking the Lagragean corresponding to (15), we obtain the following Euclidean action

$$S_{eucl} = \frac{1}{2}\tilde{S}\sqrt{\frac{\rho}{2\tilde{\delta}}} \int\int dxdy \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right], \quad (17)$$

which describes a 2D classical system, where the coupling constant $K = \sqrt{(2\tilde{\delta})/(\rho\tilde{S}^2)}$ plays the role of the temperature in the classical model. In the quantum 1D model at $T = 0$, K is a measure of the strength of quantum fluctuations.

The action given by (17) apparently coincides with the action of the scalar field, since by a scaling we can write the action as

$$S = \frac{K}{2} \int d^2x \, (\nabla\phi)^2. \quad (18)$$

However, as pointed out by Tsvelik [9], the fact that ϕ is an angle variable leads to a more complicated behavior. He shows that, for $K > 2/\pi$, the spin pair correlation function decays as a power law, while for $K < 2/\pi$ a finite correlation length arises and the correlation functions decay exponentially at distances larger than the correlation length.

As it is well known, the classical 2D planar rotor model has a Kosterlitz-Thouless (KT) transition. The renormalization group estimate for the KT transition temperature is

$$T_{KT}/S^2 = 1.40. \quad (19)$$

Using the value of ρ at $T = 0$, which is $\rho(T = 0) = 0.49$, and using the fact that the temperature in (19) is replaced by the coupling constant K, we can get a rough estimate for D_c, the critical value for D. Our estimate gives $D_c = 0.47$. Several approximations are incorporated in this estimate: the procedure of taking the continuum limit of a Hamiltonian is not free of assumptions that introduce approximations whose extension is hard to estimate. Moreover, the estimate of D_c by the simple way sketched above, depends strongly on a precise knowledge of T_{KT}. The numerical calculation performed by Chen et al. [16] for the antiferromagnetic easy plane chain gave $D_c = 0.44$. Thus, our estimate for D_c is in good accordance to the value obtained by Chen et al., in despite of all approximations done.

It is well known that, for small K, the model has quasi-long-range spin order with power law decay of spin correlations. We may refer to this phase as a Tomonaga-Luttinger liquid. For the large K phase, the spin correlations decay exponentially in imaginary time, and therefore the phase has a spin gap. The transition between the small and large K phases is described by the standard KT theory of the classical model in 2D.

We remark that our results differ from former calculations by the presence of quantum fluctuations in the stiffness: ρ is renormalized by the effects of quantum fluctuations, and, therefore, we got a better agreement with the numerical calculation. This is important if we want to make comparison with numerical or experimental data.

At this point, we would like to remember the difference between the planar rotor and the XY models. The planar rotor model has only two spin components while the XY model has three. These two models have also different KT transition temperatures.

4 Two-dimensional quantum model

We will now consider the Hamiltonian (11) in 2D. Our aim here is to study the KT phase transition using the SCHA. Supposing that, up to the transition temperature, we may consider

$$|\ \phi_{\mathbf{r+a}} - \phi_{\mathbf{r}}\ | \ll 1 \qquad \text{and} \qquad (S_r^z)^2 \ll 1, \tag{20}$$

where $\mathbf{r}$ specifies a site of the 2D lattice and $\mathbf{a}$ denotes one of its four nearest neighbors, we can use the formalism presented in Section 2. The Fourier transform of (11) in 2D is given by

$$\mathcal{H}_{FT} = 2J \sum_{\mathbf{k}} \left\{ \tilde{S}^2 \rho \left[1 - \Gamma(\mathbf{k})\right] \phi_{\mathbf{k}} \phi_{-\mathbf{k}} + (1+\delta)\, S_{\mathbf{k}}^z S_{-\mathbf{k}}^z \right\}, \tag{21}$$

where, in 2D, the anisotropy parameter is $\delta = D/2J$, $\Gamma(\mathbf{k}) = 1/2(\cos k_x + \cos k_y)$, and the expression for the stiffness is given by

$$\rho = \left[1 - \frac{I(\rho,T)}{4J\tilde{S}^2(1+\delta)}\right] \exp\left[-\frac{I(\rho,T)}{4J\tilde{S}^2\rho}\right]. \tag{22}$$

In (22), $I(\rho,T)$ is a two-dimensional integral defined as

$$I(\rho,T) = \frac{1}{\pi^2} \int_0^{\pi} dk_x \int_0^{\pi} dk_y \omega(\mathbf{k}) \coth\left(\frac{\omega(\mathbf{k})}{2T}\right), \tag{23}$$

where we have set $k_B = 1$. The dispersion relation $\omega(\mathbf{k})$ is given by

$$\omega(\mathbf{k}) = 2J\tilde{S}\sqrt{\rho[1 - \Gamma(\mathbf{k})](1+\delta)}. \tag{24}$$

Renormalization group analysis [25] shows that, at T_{KT}, the stiffness should exhibit a universal jump which is given by $2T_{KY}/\pi J\tilde{S}^2$. The KT temperature for the XY model, given by Hamiltonian (21), can then be determined by the crossing between the $\rho(T)$ curve, calculated using (22), and the line $\eta = 2T/\pi J\tilde{S}^2$. The transition temperature calculated using this approach is shown in Figure 3 as a function of $\delta = D/2J$; here, we have used $J = 1$ and $S = 1$. At $\delta = 0$, we have $T_{KT}^{XY}/J\tilde{S}^2 = 1.01$. The transition temperature T_{KT}^{XY} decreases as δ increases, and, at $\delta_c = 3.60$, the line of phase transition terminates: we identify this value of the anisotropy as the quantum critical point.

At $T \neq 0$, the two-dimensional system undergoes a KT transition. However, at $T = 0K$, for $\delta > \delta_c$, the quantum fluctuations are then so large that the system does not order and the quantum critical point should be characterized by the exponents of the 3D XY model.

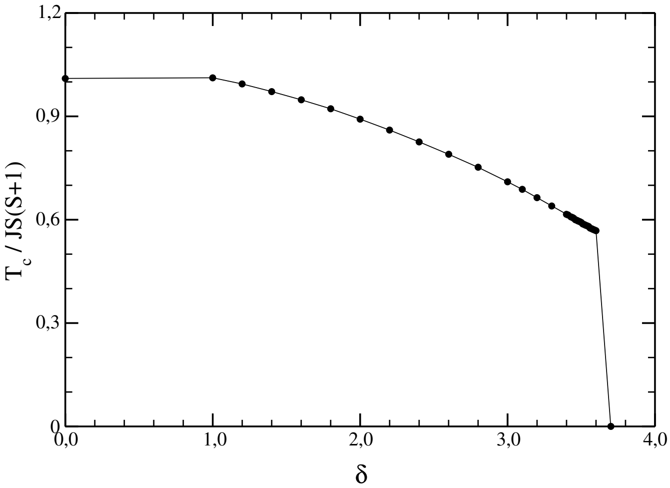

Fig. 3. Transition temperature for the 2D model described by Hamiltonian (11) as a function of the anisotropy δ.

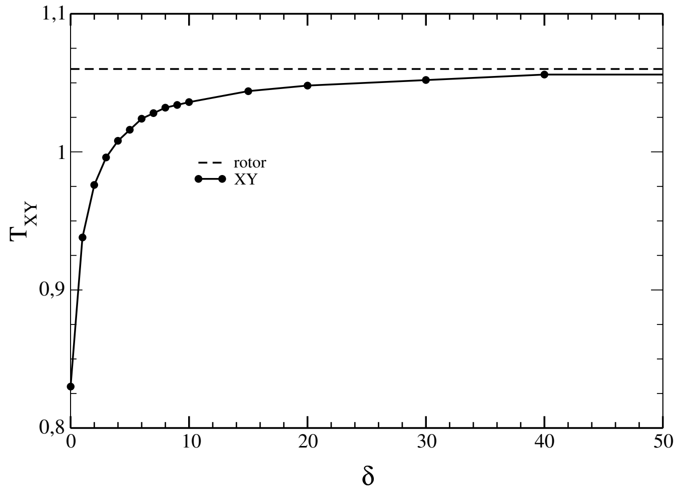

Fig. 4. The Kosterlitz-Thouless transition temperature as a function of the anisotropy δ for the classical version of Hamiltonian (11). The continuous line gives the result for the XY classical model while the dashed line corresponds to the transition temperature for the classical planar rotor model.

It is interesting to compare the behavior of the transition temperature for the quantum and classical versions of (21) as the easy-axis anisotropy D increases. If we take $D = 0$ in (21), we have the classical XY model, while the limit $D \to \infty$ gives the planar rotator model. Taking the classical limit of (22) corresponds to use $I(\rho,T) = 2T$ because, in this limit, $\coth(\omega(\mathbf{k}/2T) = 2T/\omega(\mathbf{k})$. The continuous line shown in Figure 4 gives T_{KT}/JS^2 as a function of δ for the classical case; the behavior is quite different from the observed in the quantum case (Fig. 3). For the classical model, the transition temperature of Hamiltonian (21) increases with δ, and, for δ large enough, it approaches the transition temperature for the planar rotator model, represented by the dashed line of Figure 4. The results displayed in Figure 4 were obtained taking $J = 1$ and $S = 1$.

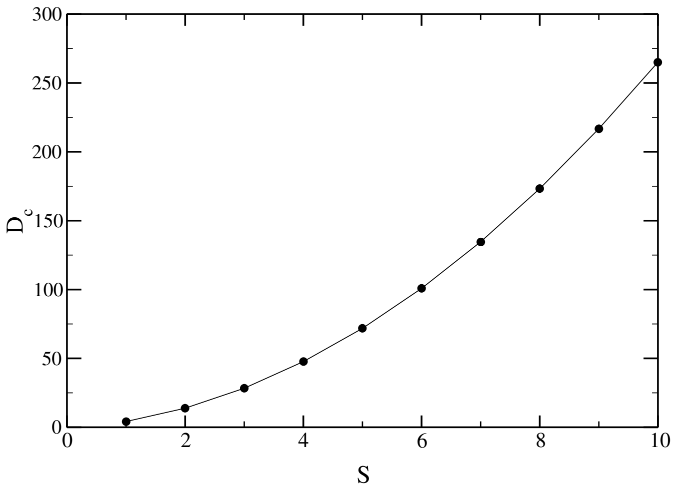

Fig. 5. The critical anisotropy for the quantum model (11) as a function of the spin S.

However, the classical and quantum versions of any Hamiltonian should show similar behaviors for large spin values. In order to understand the different behavior of $T_{KT} \times \delta$ in each case, quantum and classical, we show in Figure 5, an estimate of the critical anisotropy as a function of S. As we see, δ_c increases rapidly with S ($D_c \propto S^{1.8}$) showing the crossing from classical to quantum regime.

5 Conclusions

In this work, we have studied the one-dimensional antiferromagnet in a staggered magnetic field h using the self-consistent harmonic approximation. Our result gives a gap induced by the field depending on h according to $\Delta \propto h^{0.67}$. This behavior is quite different from the one predicted by standard spin-wave approximation ($\Delta \propto h^{1/2}$) suggesting that the 1D quantum fluctuations may be responsible for this change of behavior. However, up to our knowledge, there is no theoretical or experimental prediction for the behavior of the induced gap as a function of the applied field for the XY model.

We have also investigated the quantum phase transition of Hamiltonian (12) using a path integral representation. The value of the critical parameter, the anisotropy D, is also evaluated and is in good agreement with a numerical calculation performed by Chen et al. [16].

The SCHA is used in the study of the transition temperature of two-dimensional ferromagnetic model described by (12). It is shown that the transition temperature decreases with the single-site anisotropy $\delta = D/2J$. The line of the phase transition terminates at $\delta = 3.60$ signaling a quantum critical point. It is important to notice that the behaviors of T_{KT} for the quantum (Fig. 3) and classical models (Fig. 4) are remarkably different. For the classical model, T_{KT} does not go to zero as δ increases; instead, it approaches the value predicted for the planar rotor model. In order to understand the connection between the quantum and classical behaviors of Hamiltonian (12), we investigate the dependence of D_c with the spin S. It is shown that D_c increases rapidly with S showing that, for large spin values, the quantum and classical behaviors merge.

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